Stability and Performance Analysis of Spatially Invariant Systems with Networked Communication

Introduction

Many systems consist of interconnections of similar subsystems that only interact with their nearest neighbors [1]. Building upon our preliminary work [2], we consider such interconnections, consisting of an infinite number of spatially invariant subsystems interconnected (partly) through packet-based communication networks. These local networks are subjected to time-varying transmission intervals. Tractable local conditions are derived leading to a maximally allowable transmission interval (MATI) for all individual local communication networks such that global stability properties for the overall system (in the sense of uniform global exponential stability (UGES) or $L_p$-stability) are guaranteed.

System description: A hybrid systems approach

We consider the two system configurations of Fig. 1, which both lead to the same hybrid system description

\[
\mathcal{H}(s) = \begin{cases} 
\xi(s) = F(\xi, d)(s), & \tau(s) \in [0, \tau_{\text{MATI}}] \\
\xi^+(s) \in G(\xi(s)), & \tau(s) \in [\delta, \infty) \end{cases}
\]

where $\xi(s)$ is the state of the subsystem, $\tau(s)$ is a timer, and $s$ indicates the spatial coordinate. We consider $s$ to be in the set of integers $\mathbb{Z}$, which captures the infinite spatial extent.

Figure 1: Interconnected networked system configurations. The overall system is the infinite interconnection of subsystems $\mathcal{H}(s)$, $s \in \mathbb{Z}$.

Stability and performance analysis

Research goal:

Find a MATI ($\tau_{\text{MATI}}$) for all local networks $\mathcal{N}(s), s \in \mathbb{Z}$, such that UGES or $L_p$-stability is guaranteed.

For the general setup of (1), local Lyapunov-based conditions (local in the sense that they only involve the local dynamics of one subsystem in the interconnection and information about the local communication network) are obtained which lead to a bound on $\tau_{\text{MATI}}$ and allow for the construction of a global Lyapunov function such that UGES or $L_p$-stability is guaranteed for the overall system.

Example: A string of vehicles

We consider an infinite string of spatially invariant vehicles as in Fig. 2. To maintain a constant time headway $h$ (time between the vehicles), wireless communication is used to transmit velocity, acceleration and jerk data from the vehicle at $s \in \mathbb{Z}$ to the following vehicle at $s + 1$.

Figure 2: A string of vehicles.

Using the obtained stability analysis, values for $\tau_{\text{MATI}}$ guaranteeing UGES of the infinite string are obtained. These are shown in Fig. 3 for various time headways.

Figure 3: Upper bounds for $\tau_{\text{MATI}}$ guaranteeing UGES.

References
