M.Sc. Project

**Goal-Oriented Adaptivity for Stochastic Finite Element Simulations**

TU/e, Mechanical Engineering, Multiscale Engineering Fluid Dynamics  
TU/e, Mechanical Engineering, Numerical Methods in Engineering

**Background**
For the purpose of analysis, engineering structures are often idealized. Dimensions are assumed to match the exact specifications, and material properties are considered perfectly homogeneous. Although these simplifications are convenient from the stand point of analysis, reality paints a different picture. Manufacturing tolerances and structural damage can dramatically affect the performance of a structure, material flaws can lead to failure, and unpredictable environmental conditions can lead to critical loads.

Numerical methods have been developed to incorporate these uncertain imperfections in analyses. These methods can be viewed upon as an extension of the deterministic Finite Element Method (FEM) and are referred to as Stochastic Finite Element Methods (SFEM). In contrast to the deterministic simulations, properties are no longer perfectly known, but are represented by random fields. When discretized, a random field represents uncertain properties in terms of a finite number of random variables. As these random variables make the input to the simulation stochastic, also the computed response becomes stochastic. A method which gained particular success in approximating this random output is the so-called Spectral SFEM. In this method the random space is discretized by a finite number of spectral basis functions, and Galerkin's method is used to approximate the random response.

![Schematic representation of a layered composite material (left). When zooming in on the fiber-epoxy layers, a chaotic distribution of the fibers is observed (right). This micro-structural randomness can significantly influence the macroscopic failure behavior.](image)

In general, the biggest disadvantage of the Spectral SFEM is the involved computational effort. As the random input often requires a relatively large number of independent random variables, the stochastic space is high-dimensional. This holds in particular when accurate quantitative results are to be obtained. Fortunately, however, engineering interest is often on statistical quantities of the response, such as the means, standard deviations, and correlation coefficients. Numerical techniques exist that exploit this selective interest by adaptively changing the basis to optimally predict the goal quantities. These goal-oriented adaptivity approaches have proved to be effective for a variety of applications, and are promising to drastically reduce the computational effort of the Spectral SFEM.

**Goal**
To study the possibilities of applying goal-oriented adaptivity to the Spectral SFEM. This study will be done on a simple, one-dimensional, beam problem. The assignment on one hand requires understanding of the mathematics behind the adaptivity scheme, and on the other hand requires numerical implementation of the problem.

**Further information**
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