Feedback, sensitivity, and complexity

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Celestial mechanics:
another illustration of complexity

A common theme:
How to model the interconnection between the tiny and the large?

Lecture 1: the local and the global. A control theorist viewpoint

Lecture 2: the complexity of sensitivity analysis across scales

Lecture 3: a simple paradigm for robust control across scales
Neuronal excitability is very well understood

Solution of membrane equation (circa 1952) – using a desk calculator!

Recording from giant squid axon (circa 1952)

Hodgkin & Huxley, J Physiol. (1952)

Part I. The local
A local anecdote (1990)

Nonlinear control...

“But science is linear, isn’t?”

Theme 1 (science)

Local
Linear
Simple
methods are our only window onto a

global
nonlinear
complex

world.
A local anecdote (2000)

The essence of feedback?

“Feedback linearizes!”

Theme 2 (engineering)

Zooming principles are key to the efficiency of local windows
Two “local” anecdotes (changing the resolution)

A linearization principle

*Newton discovered a way of solving any equations (...). He regarded this discovery as his most important achievement (...).* (V. I. Arnold)

A zooming principle

*Black’s patent application for the negative feedback amplifier took some nine years to get approved (...).*

The principle of linearization

Nearby behaviors can be studied by local methods

Earth-moon-sun behavior as a nearby behavior of the earth-sun behavior
The feedback principle of localization

Interconnections change the meaning of ‘local’

By late 1927, Black's prototype negative feedback amplifier "achieved a distortion reduction of 100,000 to 1 with a frequency range extending from 4 to 45 kHz."

The local

- The linearization principle: nearby behaviors can be studied by linear methods. A foundation of modern science.

- The feedback principle: interconnections change the resolution of local. A foundation of modern engineering.
Part II. *The global*

The world is complex …

Behaviors: what we wish to understand (through mathematical models)

‘Laws that relate signals’ (J.C. Willems)

Complex behaviors: those that we do not understand (yet)
Examples of (complex) behaviors

- **Our economy**
- **Our brain**
- **Our climate**

A popular complex behavior for a 1990 engineering student
A hub of complexity in a simple world

\[ \omega(= \dot{\theta}) \]

Strogatz chaotic water wheel.

\[ I \dot{\omega} = -\nu \omega + g \Re z \]
\[ \dot{z} = -(K + j \omega)z + q \]

A simple ‘exact’ law, yet an unpredictable future

Simple laws may determine ‘erratic’ behaviors

Enormous impact on the scientific community: quest for the “simple law” that determines “complex behaviors”

(stock markets, heart rate variability, epileptic seizures, positive emotions, ...)

A hub of complexity in a simple world
From the abstract: 188 participants (...) provided daily reports of experienced positive and negative emotions over 28 days. Results showed that the mean ratio of positive to negative effect was above 2.9 for individuals classified as flourishing and below that threshold for those not flourishing.

The water wheel as an open system

<table>
<thead>
<tr>
<th>Flow</th>
<th>Low flow</th>
<th>Medium flow</th>
<th>High flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Behavior</td>
<td>Linear exogenous</td>
<td>Resonant sensitive</td>
<td>Switching endogenous</td>
</tr>
<tr>
<td>q (flow)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ω (velocity)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ B = \begin{bmatrix} q \end{bmatrix} \]
Deterministic chaos

Simple laws may determine ‘erratic’ behaviors

Whether a law and an initial condition are sufficient for prediction is a question that can be formulated for closed systems only.

Whether a behavior is sensitive (or resonant) is a valid question for open systems. It does not imply nor require ‘chaos’.

Feedback tunes sensitivity

Negative feedback | Balanced feedback | Positive feedback
Linear behavior | resonant behavior | Switching behavior
exogenous | sensitive | endogenous

damping = negative feedback balances
gravity = positive feedback
Interconnections: Push for theory of open systems

The global

Complexity is a temporary and evolving concept

Behaviors as closed systems

- Laws determine behaviors
- Observing our universe (celestial mechanics)

Behaviors as open systems

- Interconnections shape behaviors
- Interacting with our universe (global warming; the brain)
Part III. *Encounters*  
*(local windows on the global)*

Feedback glocalizes

**Negative feedback balanced by positive feedback**
The negative feedback amplifier ‘linearizes’

$V = \text{sat}_{1}(I - KV) \equiv V = \text{sat}_{\frac{1}{1+K}}(I)$

Sensitivity domain is spread by negative feedback
(The essence of control theory)

The positive feedback amplifier ‘quantizes’

$V = \text{sat}_{1}(I + KV) \equiv V = \begin{cases} +1 & I \geq -1 - K \\ -1 & I \leq K - 1 \end{cases}$

Sensitivity domain is spread by positive feedback

Hysteretic behavior: memory, ON-OFF devices
(The essence of digital technology)
The feedback amplifier principle

- Negative feedback linearizes
- Continuous behavior
- Analog technology
- Exogenous
  (output primarily reflects the input)

- Positive feedback quantizes
- On-Off behavior
- Digital technology
- Endogenous
  (output primarily reflects memory of the past)

The success of negative feedback turned positive feedback into history.

(Tucker, 1972)

Positive feedback, as something deliberately intended, is nowadays of much less significance than negative feedback, which forms the basis of control systems. In terms of mechanical systems, negative feedback in the form of governors was important long before positive feedback was recognized either implicitly or explicitly. But in electronic circuits it was the other way round; positive feedback for a couple of decades from 1912 reigned supreme, and negative feedback was something ‘invented’ for electronic systems around 1930.

(Tucker, 1972)
A historical hint: the rise of cybernetics

1920... 1927

Ashby

Black amplifier

1952

Wiener

Hodgkin & Huxley

Turing

The digital age turned cybernetics into history

1952

The digital age

2015

Crick & Watson

Von Neumann

computer
internet

AGCTCAAGCTGCTGTGGGTGATCTGCTCCTCAAACCCACAGCCTGGGTAGCAAG
AGGAACCTGTGACTGCTGGAAATCTCCTTTTCTCTGCTGGTGAAG
GACGAGACATGACTTTGGATTTTCTCCACAGGAGTGGTTGCAACAGATTTCCAAAGGCT
GAACATCCCTGTCTCCCTAGTGGATGGATCTCCAAGCTCTCTCCGACACA
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TACCCCTGATGAGCCTGGAGCTTGGAGAATGGGCTTGGAGAGG
ACTCTCATTGAAAGAAGAATAACCCCTTGTGCTGAGGTGTTGCAACAGCA
GAAATGTAGAGATTTTCTTTGTCACAAACATTGCAAGAAAAAGTTTAAGAAGAG
GGATTGA, TGTGATCTGCCAACACACAGCCTGGTAGCAGGAAGACTCCTGAGATGC
TCCTGCGACAGGAGGAGATCTCTCCTTTTCTCGCTGTTGAGGACGACATGACT
A methodological hint

There is no theory of feedback in the digital age

Recurrence is usually associated to intractability.

We go around this limitation by substituting feedforward to feedback

Examples abound in information theory, signal processing, machine learning, graphical modeling, automata theory.

Feedback: the great absent of mathematical science

Occurrences of the word “feedback” are exceptional throughout physics, mathematics, and computer science. Usually associated to “positive feedback” (autocatalysis, ...)

revealing statistics:

3 occurrences (positive feedback) 3 occurrences (positive feedback)
An architecture for multiresolution behaviors

Complexity, feedback, and sensitivity

Open and closed systems

The analysis tools of complex behaviours are inherited from questions raised by celestial mechanics. Those questions are formulated in the language of closed systems. Instead, current questions pertaining to complexity are about large interconnections of open systems.

Feedback

Feedback is central to study interconnections. Feedback is essential to localisation. Localisation is essential to tractability. Feedback is an essential bridge between analog and digital behaviours.

Sensitivity

Complex systems are about interconnections between the tiny and the large. Sensitivity analysis is a central analysis tool of control theory.
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