Data-driven modelling in dynamic networks

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ICMS Winterschool, TU/Eindhoven, 17 February 2017
Introduction – dynamic networks

Decentralized process control

Power grid

Metabolic network

Distributed control (robotic networks)

Stock market

Introduction – dynamic networks

Drivers for **data-processing / data-analytics**

Providing the tools for **online**
  • Model estimation / calibration / adaptation

To accurate perform online model-based **X**:
  • Monitoring
  • Diagnosis and fault detection
  • Control and optimization
  • Predictive maintenance
  • Controller reconfiguration
  • ...........

Turn large amounts of (relatively inexpensive) data into process/economic value
Industry 4.0 – process operations aspects

From isolated (statically) optimized units to

- integrated chains/networks of production units,
- fully automated, high level of sensing/actuation,
- data and product flows across classical (company) borders (suppliers, customers, energy grid)
- modular build-up
- continuously monitored for control, optimization, (predictive) maintenance, analysis, ......
- adapting to changing circumstances (process and market conditions), and learning
- economically optimized
- supervised by new-generation HMI technology and operators

Dynamical systems are considered to have a more complex structure:

- distributed control system (1d-cascade)
- dynamic network

(distributed MPC, multi-agent systems, biological networks, smart grids,…)

For on-line monitoring / control / diagnosis it is attractive to be able to identify:
- (changing) dynamics of modules in the network
- (changing) interconnection structure
The classical (multivariable) identification problems: 

Identify a plant model $\hat{G}$ on the basis of measured signals $u$, $y$ (and possibly $r$)

- We have to move from fixed and known configuration to deal with and exploit structure in the problem.

[ workforce(Ljung (1999)) ]
Introduction – Dynamic network identification

Some modules may be known (e.g. controllers)

- $r_i$: external excitation
- $v_i$: process noise
- $w_i$: node signal
Introduction – Dynamic network identification

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Introduction – Dynamic network identification

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Some modules may be known (e.g. controllers)
Introduction – relevant identification questions

Identification of a single (local) module?

Where to place sensors and actuators for optimal accuracy?

How to utilize known structure/topology and known modules?

Can we identify the topology?

Is the full network identifiable?

Towards dynamic network identification

- Basic identification tools: direct and projection
  - From closed-loop to dynamic networks
- Single module identification - consistency
  - full MISO models
  - predictor input selection
- Example of decentralized control
- Additional results and discussion
Methods for closed-loop identification

1. Direct method

Relying on full-order noise modelling;
Prediction error
\[ \varepsilon(t, \theta) = H(\theta)^{-1}[y(t) - G(\theta)u(t)] \]
Using only signals \( u \) and \( y \), discarding \( r \)
\[ \hat{\theta}_N = \arg \min_{\theta} \frac{1}{N} \sum_{t=1}^{N} \varepsilon(t, \theta)^2 \]

2. Projection/two-stage/IV method

Relying on measured external excitation \( r \)
\[ \varepsilon(t, \theta) = H(\rho)^{-1}[y(t) - G(\theta)u^r(t)] \]
with \( u^r \) the signal \( u \) projected onto \( r \)
Similar least squares criterion.

Plant representation
\[ y(t) = G_0 u(t) + H_0 e(t) \]
e white noise
\( r \) and \( v \) uncorrelated
Methods for closed-loop identification

1. **Direct method**  [Ljung, 1987]

   Consistent estimate of \( \{G_0, H_0\} \)
   provided that \( u \) is sufficiently exciting

2. **Projection/two-stage/IV method**  [Van den Hof & Schrama, 1993]

   Consistent estimate of \( G_0 \)
   provided that \( u^r \) is sufficiently exciting

\[
y(t) = G_0 u(t) + H_0 e(t)
\]

\( e \) is white noise

\( r \) and \( v \) uncorrelated
Assumptions:

- Total of $L$ nodes
- Network is well-posed
  
  \[
  I - G^0 \quad \text{causally invertible}
  \]
- Stable (all signals bounded)
- All $w_m, m = 1, \cdots L$, measured, as well as all present $r_m$
- Modules may be unstable
Identifying a module

Options for identifying a module:

- Identify the full MIMO system:
  \[ w = (I - G^0)^{-1}[r + v] \]
  from measured \( r \) and \( w \).
  
  Global approach with “standard” tools

- Identify a local (set of) module(s)
  from a (sub)set of measured \( r_k \) and \( w_L \)

  Local approach with “new” tools and structural conditions
Identifying a module

How to identify a module:

Suppose we are interested in $G_{21}^0$

Can it be identified from measured input $w_1$ and output $w_2$?

Typically bias will occur due to “neglecting” the rest of the network

- Non-modelled disturbances on $w_2$ can create problems
- The observed transfer between $w_1$ and $w_2$ is not necessarily $G_{21}^0$
How to identify a module:

Two approaches for finding $G_{21}^0$

- **Full MISO approach:** Include all node signals that directly map into $w_2$ in an input vector, and identify a MISO model.

- **Predictor input selection:** Formulate conditions for checking the sufficiency of set of nodes to include as inputs in a MISO model.
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- Additional results and discussion
Full MISO models – Direct method

- Module of interest: $G_{ji}^0$
- Separate the modules $G_{jk}^0$ into known modules: $G_{jk}^0$, $k \in \mathcal{K}_j$
  and unknown modules: $G_{jk}^0$, $k \in \mathcal{U}_j$
- Determine: $\bar{w}_j(t) = w_j(t) - r_j(t) - \sum_{k \in \mathcal{K}_j} G_{jk}^0(q)w_k(t)$
- Prediction error: $\epsilon(t, \theta) = H_j(\theta)^{-1}[\bar{w}_j(t) - \sum_{k \in \mathcal{U}_j} G_{jk}(\theta)w_k(t)]$

Simultaneous identification of $G_{jk}^0$, $k \in \mathcal{U}_j$ and $H_j^0$

Consistent estimates if $\{w_k\}_{k \in \mathcal{U}_j}$ sufficiently exciting, and $\Phi_v(\omega)$ diagonal

[P.M.J. Van den Hof et al., Automatica, October 2013]
Network Identification – Projection method

Algorithm:

- Find an $r_m$ with a path to $w_i$ such that $w_i^{r_m}$ is present.
- Construct:
  $$\tilde{w}_j = w_j - r_j - \sum_{k \in \mathcal{K}_j} G_{jk}^0(q)w_k$$

  known terms

- Prediction error:
  $$\varepsilon(t, \theta) = H_j(\rho)^{-1}\left[\tilde{w}_j - \sum_{k \in \mathcal{U}_i} G_{jk}(\theta)w_k^{r_m}\right]$$

  where all inputs $k \in \mathcal{U}_i \subseteq \mathcal{U}_j$ are considered that are correlated to $r_m$

  **Consistent identification of $G_{jk}^0$, $k \in \mathcal{U}_i$ provided that $\{w_k^{r_m}\}_{k \in \mathcal{U}_i}$ sufficiently exciting**

- This extends to multiple signals $r_m$

[P.M.J. Van den Hof et al., *Automatica*, October 2013]
Network Identification – Two-stage method

Example

• External signal $r_1$

• Input nodes to $w_2$ that are correlated with $r_1$: $w_1, w_6, w_7, w_3$

• So 4 input, 1 output problem

• Projected inputs will generally not be sufficiently exciting (we need 4 independent sources)

• Include $r_4, r_5$ and $r_8$ as external signals

• Input nodes remain the same as for direct method
Network Identification – Full MISO models

Observations:

• Consistent identification of single transfers is possible, dependent on network topology and reference excitation
• Choice between estimating accurate noise models (direct method) and utilizing reference excitation (projection method)
• Excitation conditions on (projected) input signals can be limiting
• Network topology conditions on $r_m$ can simply be checked by tools from graph theory
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Predictor input selection

• So far: predictor input choice not very flexible

• What if some signals are hard (expensive) to measure?

• What if we would like to have flexibility in placing sensors?

• Can we formulate (more relaxed) conditions on nodes to be measured, for allowing a consistent module estimate?
There are two basic mechanisms that “deteriorate” the transfer $G^0_{ji}$ when nodes are removed:

1. Parallel paths
2. Loops around $w_j$

To maintain $G^0_{ji}$ these should be “blocked” by measured nodes (predictor inputs)
**Objective:** obtain an estimate of $G_{ji}^0$

Consistent estimates of $G_{ji}^0$ are possible if:

1. $w_i$ is included as predictor input
2. Each parallel path from $w_i \rightarrow w_j$ passes through a node chosen as predictor input
3. Each loop from $w_j \rightarrow w_j$ passes through a node chosen as predictor input
Example with predictor input conditions

**Objective:** Estimate $G_{21}^0$.

**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

**Conclude:** include $w_1$ and ... as predictor inputs
Example with predictor input conditions

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Example with predictor input conditions

**Objective:** Estimate $G_{21}^0$.

**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2 \Rightarrow$ Include $w_6$ in predictor
- $w_2 \rightarrow w_2$

**Conclude:** include $w_1$, $w_6$ and ... as predictor inputs
Example with predictor input conditions

**Objective:** Estimate $G^0_{21}$.

**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

**Conclude:** include $w_1, w_6$ and … as predictor inputs
Example with predictor input conditions

**Objective:** Estimate $G_{21}^0$.

**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

**Conclude:** include $w_1, w_6$ and $\ldots$ as predictor inputs
Example with predictor input conditions

**Objective:** Estimate $G_2^0$.

**Conditions:** Include variable on every path
- $w_1 \rightarrow w_2 \implies$ Include $w_6$ in predictor
- $w_2 \rightarrow w_2 \implies$ Include $w_3$ in predictor

**Conclude:** include $w_1$, $w_6$ and $w_3$ as predictor inputs
**Predictor input selection**

**Result**

The consistency results of both direct and projection method remain valid if

- the set $\mathcal{D}_j$ of predictor inputs satisfies the formulated conditions
- For the direct method: there are no confounding variables
- For the projection method: no excitation signal used for projection, has a path to $w_j$ that does not pass through a node in $\mathcal{D}_j$

In the “full” MISO case: consistent estimates of all $G_{jk}^0$, $k \in \mathcal{U}_j$

In the “selected” predictor input case: consistent estimates of $G_{ji}^0$

Predictor input selection

For direct method: $w_7$ is a confounding variable and needs to be included.
For projection method: no problems.
• The two conditions (parallel paths and loops on output) result from an analysis of the so-called immersed network.

• The immersed network is constructed on the basis of a reduced number of node variables only, and leaves present node signals invariant.

• Whether dynamics in the immersed network is invariant can be verified with the graph theory/tools of separating sets.

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Example decentralized MPC; 2 interconnected MPC loops

Target:
Identify interaction dynamics
\[ G_{21}, G_{12} \]

Addressed by
Gudi & Rawlings (2006)
for the situation \[ G_{12} = 0 \]
(no cycles)

Example decentralized control

Case of Gudi & Rawlings (2006):

Target:
Identify interaction dynamics $G_{21}$

\[
\begin{align*}
    u_2 &= R_2^i r_2 - R_2^i G_{21} u_1 - R_2^i v_2 \\
    y_2 &= S_2^0 G_2 C_2 r_2 + S_2^0 G_{21} u_1 + S_2^0 v_2
\end{align*}
\]

Options:
1. Identify from $(r_2, u_1) \rightarrow u_2$
   and find $G_{21}$ by taking the quotient of the two models

2. a) Identify $R_2^i$ from $r_2 \rightarrow u_2$

   Simulate: $u_f = (R_2^i)^{-1} u_2$

   b) Identify $G_{21}$ from $u_1 \rightarrow u_f$

Excitation through dither signals on $r_2$ and $u_1$
According to network results (input selection):

\[ y_2 = G_{21}u_1 + G_2u_2 + v_2 \]

Estimate 2-input 1-output model:
\[(u_1, u_2) \rightarrow y_2\]
provides consistent estimate of \( G_{21} \) through both direct and projection method

- Excitation properties of signals remain important:
- Direct method utilizes excitation through noise signals \( v_1, v_2 \)
The more general situation (cyclic connection):

\[ y_1 = G_1 u_1 + G_{12} u_2 + v_1 \]
\[ y_2 = G_{21} u_1 + G_2 u_2 + v_2 \]

Estimate 2-input 1-output models:
\[ (u_1, u_2) \rightarrow y_1 \]
\[ (u_1, u_2) \rightarrow y_2 \]
provides consistent estimates of
\[ G_{21}, G_{12} \]
together with \[ G_1, G_2 \]

If plant models \[ G_1, G_2 \] are known the situation simplifies.
Example decentralized control

Observation

Network identification results provide a formal way to handle these structured identification problems.
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Sensor noise – the errors-in-variables problem

What if node variables are measured with (sensor) noise?

- Classical (tough) problem in open-loop identification
- More simple in dynamic networks due to the presence of multiple (correlated) node signals

[A. Dankers et al., Automatica, December 2015]
Question
Can network models of a full network be distinguished from each other?

Consider: \[ T(q) = (I - G(q))^{-1} \begin{bmatrix} H(q) & R(q) \end{bmatrix} \]

mapping: \[ \begin{pmatrix} e \\ r \end{pmatrix} \rightarrow w \]

For identifiability of a model set, different network models should lead to different \( T \)’s

This puts conditions on:
- The presence of excitation signals and process noise
- The number of modules that can be parametrized

[H.H.M. Weerts et al, IFAC SYSID 2015, and IFAC ALCOSP 2016]
Discussion / Wrap-up

• So far: focus on (local) consistency results in networks with known structure and linear dynamics

• Many additional questions/topics remain:
  Variance of estimates, influenced by
  − Additional (output) measurements
  − Excitation properties

  [See e.g. work of H. Hjalmarsson, B. Wahlberg, N. Everitt, B. Günes, M. Gevers, A. Bazanella]

• Optimal sensor and actuator locations – experiment design

• Algorithms for application to large-scale systems
• **Identification of the structure/topology** addressed in the literature, in particular forms:
  • Tree-like structures (no loops)
  • Nonparametric methods (Wiener filter)
  • Mostly networks **without external excitation** and uncorrelated (white) process noises on every node

  see e.g. Materassi, Innocenti (TAC-2010), Chiuso and Pillonetto (Automatica, 2012)

• **Sparse identification** methods can be used in an identification setting to identify the topology (non-zero transfers)

• **New identifiability concepts** apply to the unique determination of a network topology

  see e.g. Goncalves & Warnick (TAC-2008), Weerts et al. (SYSID-2015).

• **Connection with decentralized/distributed control**
Acknowledgement

Co-workers:

Arne Dankers.
Harm Weerts
Xavier Bombois
Peter Heuberger
Jobert Ludlage
Mohsin Siraj
Mehdi Mansoori
Papers available at www.pvandenhof.nl/publications.htm